

ALGEBRA II



QUESTION CATALOGUE

Algebra II

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I. LOGIC

A. Logical Operations

1. Logical Equivalence

a. Original and Contrapositive

2. Which is the negation of the statement "Some rectangles are squares"?
- (1) Some rectangles are not squares.
 - (2) All rectangles are not squares.**
 - (3) All rectangles are squares.
 - (4) All squares are rectangles.
3. Which statement is logically equivalent to $\sim p \rightarrow q$?
- (1) $p \rightarrow q$
 - (2) $p \rightarrow \sim q$
 - (3) $\sim q \rightarrow p$**
 - (4) $q \rightarrow p$
4. Given: $a \rightarrow b$ and $c \rightarrow \sim b$.
Which statement is a logically valid conclusion?
- (1) $a \rightarrow c$
 - (2) $a \rightarrow \sim c$**
 - (3) $b \rightarrow c$
 - (4) $\sim b \rightarrow a$
7. Assume that the statement, "All geniuses have studied geometry," is true. Which statement must also be true?
- (1) Ron has studied geometry; therefore, Ron is a genius.
 - (2) Mary is not a genius; therefore, Mary has not studied geometry.
 - (3) Lance has not studied geometry; therefore, Lance is not a genius.**
 - (4) If Lucy studied geometry, Lucy is a genius.
8. If $A \rightarrow B$ and $\sim C \rightarrow \sim B$, then
- (1) $A \rightarrow C$**
 - (2) $A \rightarrow \sim C$
 - (3) $\sim A \rightarrow \sim C$
 - (4) $\sim A \rightarrow C$
9. Which is the negation of the statement, "Larry is old and Gary is not here"?
- (1) Larry is old and Gary is here.
 - (2) Larry is not old or Gary is here.**
 - (3) Larry is not old and Gary is not here.
 - (4) Larry is old or Gary is here.
12. Given the premises: $p \vee \sim q$
 $\sim p$

Write a logical conclusion.

$\sim q$

13. Which is logically equivalent to $\sim(\sim p \wedge q)$?
- (1) $p \wedge \sim q$
 - (2) $p \vee \sim q$**
 - (3) $\sim p \wedge \sim q$
 - (4) $\sim p \vee \sim q$
14. Given the true statements, "If Paul catches fish today, then he will give me some," and "Paul will give me some fish."
- Which statement must be true?
- (1) Paul will not give me some fish.
 - (2) Paul will not catch some fish today.
 - (3) Paul will catch some fish today.
 - (4) No conclusion is possible.**
20. If $F \rightarrow \sim G$ and G , then which is true?
- (1) $\sim F$**
 - (2) $\sim G$
 - (3) F
 - (4) no conclusion is possible
21. Which is the negation of the statement, "I like hamburgers and I do not like hot dogs"?
- (1) I like hamburgers or I do not like hot dogs.**
 - (2) I do not like hamburgers and I like hot dogs.
 - (3) I do not like hamburgers or I like hot dogs.
 - (4) I do not like hamburgers and I do not like hot dogs.
24. Write another statement that is logically equivalent to $\sim(p \wedge \sim q)$.
- $\sim p \vee q$ or $p \rightarrow q$ or $\sim q \rightarrow \sim p$
25. Which is *always* false?
- (1) $p \wedge \sim p$**
 - (2) $p \vee \sim p$
 - (3) $\sim(p \vee q)$
 - (4) $\sim(p \rightarrow q)$
26. Given the true statements: "If a boy plays high school football, he must be passing in three subjects," and "Bob is not passing three subjects." It follows that
- (1) Bob plays on the football team
 - (2) Bob does not play on the football team**
 - (3) few boys try out for the team
 - (4) no conclusion can be reached

II. ALGEBRA

A. Numbers, Sets, Functions, Systems, and Operations

302. Using the accompanying table, find y if $a * y = c * d$.

*	a	b	c	d
a	b	c	d	a
b	c	d	a	b
c	d	a	b	c
d	a	b	c	d

b

306. Given the set $\{a, b, c, d\}$ and the operation Δ as shown in the accompanying table, except for the second row which has been left out. If the operation is commutative which could be the second row?

Δ	a	b	c	d
a	b	c	d	a
b				
c	d	a	b	c
d	a	b	c	d

- (1) $a b c d$ (3) $c d a b$
 (2) $b c d a$ (4) $d a b c$

310. Using the accompanying table, find the identity element for the operation $@$.

@	1	2	3	4
1	3	4	1	2
2	4	1	2	3
3	1	2	3	4
4	2	3	4	1

3

311. Compute $(z * y) * x$ in the system defined by the accompanying table.

*	x	y	z
x	x	z	z
y	z	y	z
z	x	y	x

z

539. For which operation is the set of integers *not* closed?

- (1) addition (3) multiplication
 (2) subtraction (4) **division**

2. Mathematical Systems

c. Mathematical Fields Short Answer

316. Given set $G = \{p, q, r, s\}$ with binary operation $*$ defined by the table below. Find the value of $(q * r) * (r * s)$.

*	p	q	r	s
p	q	r	s	p
q	r	s	p	q
r	s	p	q	r
s	p	q	r	s

s

317. Using the table below, solve for x if $x @ c = b$

@	a	b	c	d
a	d	a	b	c
b	a	b	c	d
c	b	c	d	a
d	c	d	a	b

a

322. Find the value of $(B \# S) \# S$ within the system defined below.

#	B	E	S	T
B	T	S	E	B
E	S	T	B	E
S	E	B	T	S
T	B	E	S	T

B

323. What is the identity element in the system defined by the table below?

#	L	U	C	K
L	K	C	U	L
U	C	K	L	U
C	U	L	K	C
K	L	U	C	K

K

1880. The set $\{0,1,-1\}$ is closed under the operation of

- (1) addition (3) subtraction
 (2) **multiplication** (4) division

III. THE COORDINATE PLANE

A. Graphing Equations

679. What is the slope of the line $2y = 3x + 12$?

$\frac{3}{2}$ or 1.5

684. Find the slope of the line which passes through the points whose coordinates are $(-2,5)$ and $(3,9)$.

$\frac{4}{5}$

694. An equation of the line which passes through the point $(0,2)$ and which has a slope of 4 is

- (1) $x = 2y + 4$ (3) $x = 4y + 2$
(2) $y = 2x + 4$ (4) $y = 4x + 2$

695. Which statement is true of the slope of the straight line that passes through the points $(5,2)$ and

$(-1,2)$?

- (1) It has no slope. (3) It has a slope of 3.
(2) **It has a slope of zero.** (4) It has a slope of $\frac{1}{3}$.

696. The vertices of triangle ABC are $A(1,1)$, $B(10,4)$, and $C(7,7)$.

a Find the slope of \overrightarrow{AB} .

b If $D(7,k)$ is a point on \overline{AB} , find k .

c Write an equation for \overrightarrow{AC} .

d If E is a point on \overline{AC} such that $\overrightarrow{DE} \parallel \overrightarrow{BC}$, find the coordinates of E .

a $\frac{1}{3}$

b 3

c $y = x$

d (5,5)

701. What is the slope of a line that passes through the points $A(2,3)$ and $B(-10,8)$?

- (1) $-\frac{5}{12}$ (3) $\frac{12}{5}$
(2) $\frac{5}{12}$ (4) $-\frac{12}{5}$

709. What is the slope of a line that passes through the points $(-1,2)$ and $(1,4)$?

1

725. Find the slope of the line joining the points $(3,-2)$ and $(4,1)$.

3

1. Linear Equations and Slopes a. Finding the Slope and Y-Intercept

710. The vertices of parallelogram $ABCD$ are $A(1,3)$, $B(6,3)$, $C(4,-1)$, $D(-1,-1)$, and diagonals \overline{AC} and \overline{BD} intersect at point E .

a Find the slope of the line passing through points B and C .

b Find the slope of the altitude drawn from vertex A to side \overline{BC} .

c Find the coordinates of point E .

d Write an equation of the line which passes through point E and is parallel to side \overline{DC} .

e Find the length of diagonal \overline{AC} .

a 2

b $-\frac{1}{2}$

c (5, 1)

2

d $y = 1$

e 5

719. What is the slope of the line that passes through the points whose coordinates are $(2,3)$ and $(-1,12)$?

- (1) $\frac{1}{3}$ (3) 3
(2) $-\frac{1}{3}$ (4) -3

722. The vertices of $\triangle ABC$ are $A(-4,1)$, $B(2,13)$, and $C(10,9)$.

a Find the length of \overline{AB} in radical form.

b Find the slope of \overline{AB} .

c Find the coordinates of point M , the midpoint of \overline{BC} .

d Write an equation of the line which is parallel to \overline{AB} and passes through point M .

e The line described in part d intersects side \overline{AC} at point D . The ratio of the length of \overline{DM} to the length of \overline{AB} is

- (1) 1:1 (3) 1:3
(2) 1:2 (4) 1:4

a $6\sqrt{5}$ or $\sqrt{180}$

b 2

c (6,11) or

$x = 6, y = 11$

d $y = 2x - 1$

e 2

IV. RATIOS AND PROPORTIONS

A. Mathematical Ratios

1036. In right triangle ABC , $\angle C$ is a right angle and $m\angle B = 60$. What is the ratio of $m\angle A$ to $m\angle B$?

$\frac{1}{2}$

1037. The altitudes of two similar triangles are in the ratio 2:3. If the perimeter of the smaller triangle is 18, find the perimeter of the larger triangle.

27

1041. If the ratio of the areas of two squares is 4:9, what is the ratio of the perimeter of the smaller square to the perimeter of the larger square?

2:3

1049. A total of \$450 is divided into equal shares. If Kate receives four shares, Kevin receives three shares, and Anna receives the remaining two shares, how much money did Kevin receive?

(1) \$100

(3) \$200

(2) \$150

(4) \$250

1050. During a recent winter, the ratio of deer to foxes was 7 to 3 in one county of New York State. If there were 210 foxes in the county, what was the number of deer in the country?

(1) 90

(3) 280

(2) 147

(4) 490

1052. At a concert, \$720 was collected for hot dogs, hamburgers, and soft drinks. All three items sold for \$ 1.00 each. Twice as many hot dogs were sold as hamburgers. Three times as many soft drinks were sold as hamburgers. The number of soft drinks sold was

(1) 120

(3) 360

(2) 240

(4) 480

1055. The profits in a business are to be shared by the three partners in the ratio of 3 to 2 to 5. The profit for the year was \$176,500. Determine the number of dollars each partner is to receive.

\$52,950, \$35,300, and \$88,250 and an appropriate method is shown, such as $3x + 2x + 5x = \$176,500$

1. Using Ratios

a. Word Problems Involving Ratios

2748. A flagpole casts a shadow 160 feet long. At the same time, a boy standing nearby who is 5 feet tall casts a shadow 20 feet long. Find the number of feet in the height of the flagpole.

40

2869. Two triangles are similar. The lengths of the sides of the smaller triangle are 3, 5, and 6, and the length of the longest side of the larger triangle is 18. What is the perimeter of the larger triangle?

(1) 14

(3) 24

(2) 18

(4) 42

2872. If the circumference of a circle is doubled, the diameter of the circle

(1) remains the same

(3) is multiplied by 4

(2) increases by 2

(4) is doubled

2875. Ninety percent of the ninth grade students at Richbartville High School take algebra. If 180 ninth grade students take algebra, how many ninth grade students do *not* take algebra?

20

2876. If the instructions for cooking a turkey state "Roast turkey at 325° for 20 minutes per pound," how many *hours* will it take to roast a 20-pound turkey at 325° ?

$6\frac{2}{3}$ or 6 hr 40 min or 6.66 or an equivalent answer.

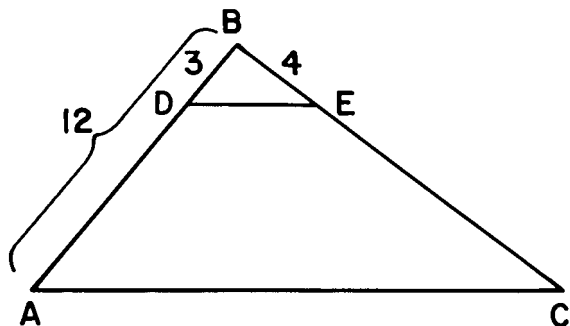
2896. At the Phoenix Surfboard Company, \$306,000 in profits was made last year. This profit was shared by the four partners in the ratio 3:3:5:7. How much *more* money did the partner with the largest share make than one of the partners with the smallest share?

\$68,000

IV. RATIOS AND PROPORTIONS

B. Similar Polygons and Right Triangles

981. In the accompanying diagram of $\triangle ABC$, $\overline{DE} \parallel \overline{AC}$. If $BD = 3$, $BE = 4$, and $AB = 12$, find the length of \overline{EC} .

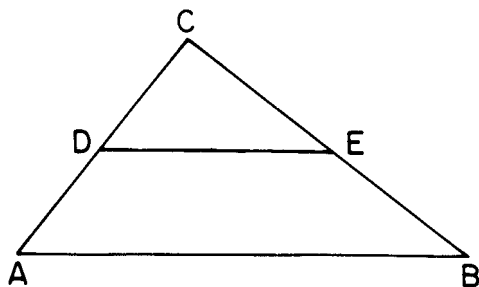


12

984. The sides of a triangle measure 3, 5, and 7. If the smallest side of a similar triangle measures 9, find its longest side.

21

985. In the accompanying diagram of $\triangle ABC$, $\overline{AD} \cong \overline{DC}$, $\overline{DE} \parallel \overline{AB}$, and $DE = 4$. Find AB .



8

986. In $\triangle ABC$, a line parallel to \overline{AB} cuts \overline{AC} at D and \overline{BC} at E . If $CD = 6$, $AC = 18$, and $BC = 27$, find EC .

9

987. The perimeter of $\triangle ABC$ is 30. Find the perimeter of the triangle formed by joining the midpoints of the sides of $\triangle ABC$.

15

1. Similar Polygons

a. Lengths of Sides of Sim Polygons

988. In $\triangle LMN$, P is a point on \overline{LM} and Q is a point on \overline{LN} such that $\overline{PQ} \parallel \overline{MN}$. If $LP = 4$, $PM = 3$, and $QN = 9$, what is the length of \overline{LQ} ?

12

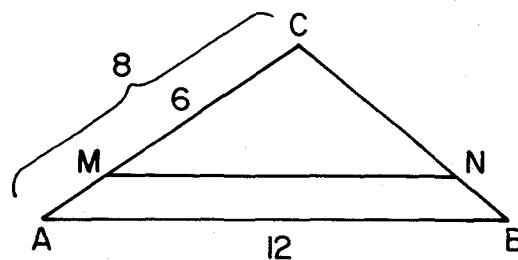
989. The ratio of the corresponding sides of two similar triangles is 7:5. Find the ratio of their perimeters.

7:5

990. The ratio of the corresponding sides of two similar polygons is 2:3. If the perimeter of the larger polygon is 27, find the perimeter of the smaller polygon.

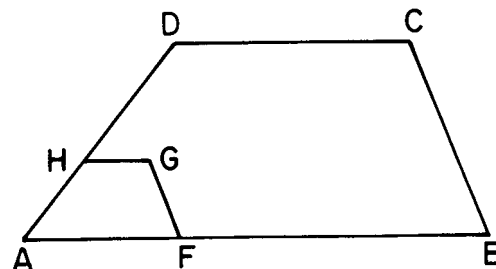
18

991. In the accompanying diagram of $\triangle ABC$, $\overline{MN} \parallel \overline{AB}$, $AC = 8$, $AB = 12$, and $CM = 6$. Find the length of \overline{MN} .



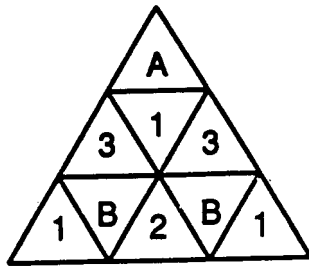
9

992. In the accompanying diagram, trapezoid $ABCD$ is similar to trapezoid $AFGH$. If $AF = 18$, $AB = 54$, and $HG = 9$, what is the length of \overline{DC} ?



27

2529. In the accompanying diagram, the triangular pad is divided into nine keys. The probability of pressing any key at random is the same.



Find the probability of pressing

- a a letter key
- b *exactly* two number keys on three random tries
- c *at least* two letter keys on three random tries

a $\frac{3}{9}$

b $\frac{4}{9}$

c $\frac{7}{27}$

2538. In a contest, the probability of the Alphas beating the Betas is $\frac{3}{5}$. The teams compete four times a season and each contest has a winner.

Find the probability that

- a the Betas win all four contests
- b each team wins two contests during the season
- c the Alphas win *at least* two contests during the season
- d the Betas win *at most* one contest during the season

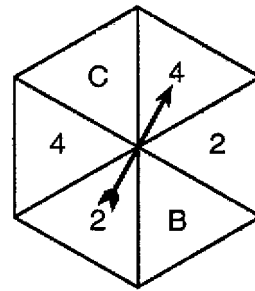
a $\frac{16}{625}$

b $\frac{216}{625}$

c $\frac{513}{625}$

d $\frac{297}{625}$

2549. In the accompanying diagram, a regular hexagon with a spinner is divided into six equal areas labeled with a letter or a number.



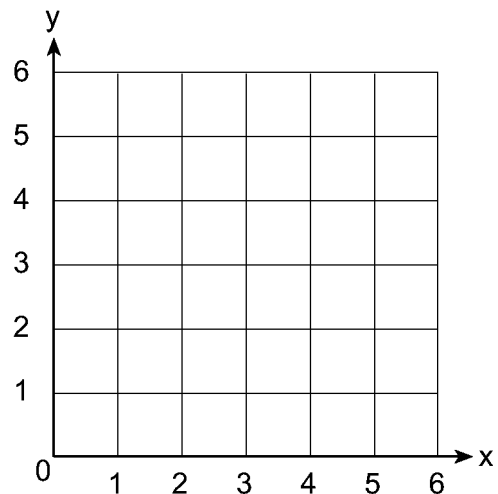
If the spinner is spun four times, find the probability that it will land in a

- (1) numbered area *at most* one time
- (2) lettered area *at least* three times

(1) $\frac{48}{81}$

(2) $\frac{9}{81}$

2864. A square dartboard is represented in the accompanying diagram. The entire dartboard is the first quadrant from $x = 0$ to 6 and from $y = 0$ to 6. A triangular region on the dartboard is enclosed by the graphs of the equations $y = 2$, $x = 6$, and $y = x$. Find the probability that a dart that randomly hits the dartboard will land in the triangular region formed by the three lines.



$\frac{8}{36}$ or $\frac{2}{9}$ or 2:9