

AP CALCULUS AB/BC



QUESTION CATALOGUE

AP Calculus

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1. Functions

1. Rational Functions

1524. The function $f(x) = \frac{x^2 + 9x - 90}{x^2 + 19x + 60}$ has a removable discontinuity at

- (A) -4 (B) 6 (C) -15 (D) 15 (E) 7

1071.

What is the domain of the function $f(x) = \frac{x^2 - 16}{x^2 + 7x + 12}$?

- (A) $(-4, 4)$ (D) $(-\infty, +\infty)$
 (B) $x \neq -3, -4$ (E) $x \neq -4, 4$
 (C) $x \neq -3, -4, 4$

1097.

The domain of the function $f(x) = \frac{1}{\sqrt{4-x}}$ is

- (A) $x \geq 0$ (D) $x \geq 4$
 (B) $x > 4$ (E) $x \leq 4$
 (C) $x < 4$

1168. The graph of $y^2 - 5y - 1 = x^2$ is a

- (A) circle (D) line
 (B) ellipse (E) parabola
 (C) hyperbola

1215. If the zeros of $f(x)$ are $x = -2$ and $x = 3$, then the zeros of $f(x/2)$ are $x =$

- (A) $-1, \frac{3}{2}$ (D) $-3, \frac{9}{2}$
 (B) $-2, 3$ (E) $-4, 6$
 (C) $-2, 6$

1244. If $f(x) = 3 - x$ and $g(x) = \sqrt{x-5}$, then $f(g(-2)) =$

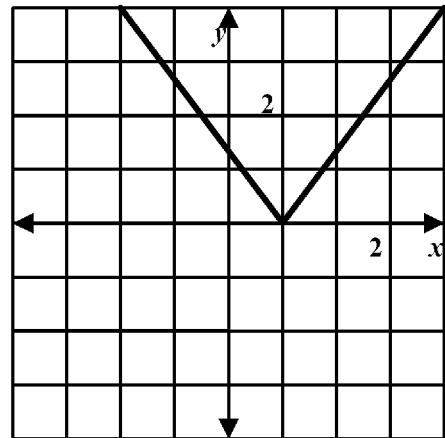
- (A) Zero (D) $\sqrt{2}$
 (B) $4 - \sqrt{7}$ (E) Undefined
 (C) $\sqrt{7}$

1525.

The function $h(x) = \frac{f(x)}{g(x)}$ is discontinuous whenever

- (A) $g(x) = 1$ (D) $g(x) = 0$
 (B) $g(x)$ is negative (E) $f(x) = g(x)$
 (C) $f(x) = 0$

1218.



Which of the following functions is represented by the graph above?

- (A) $f(x) = \left| -\frac{3}{2}x + 1 \right|$ (D) $f(x) = \frac{3}{x}x + 1$
 (B) $f(x) = \left| \frac{3}{2}x + 1 \right|$ (E) $f(x) = \frac{3}{2}x - 1$
 (C) $f(x) = -\frac{3}{2}x + 1$

1582. If $f(x) = \frac{3}{x^2 - 2}$ and $g(x) = 4x$, then $g(f(3)) =$

- (A) $3/7$ (D) $12/7$
 (B) $2/3$ (E) $14/7$
 (C) $12/3$

1726. A polynomial has a relative maximum at $(-5, 4)$, a relative minimum at $(1, -7)$ and a relative maximum at $(4, 2)$, and no other critical points. How many real zeros does the polynomial have?

- (A) 2 (D) 5
 (B) 3 (E) 6
 (C) 4

591. If h is a small negative number, then the best approximation for $\sqrt[4]{16+h}$ is

- (A) $\frac{h}{32}$ (B) $-\frac{h}{32}$ (C) $2 - \frac{h}{16}$ (D) $2 + \frac{h}{32}$ (E) $2 - \frac{h}{32}$

603. What is the linear approximation for $f(x) = \frac{1}{x^2+1}$ near $x = 1$?

- (A) $1 - \frac{1}{2}x$ (B) $\frac{1}{2} - \frac{3}{2}x$ (C) $\frac{3}{2} + \frac{1}{2}x$ (D) $\frac{1}{2} + \frac{3}{2}x$ (E) $\frac{1}{2} - \frac{1}{2}x$

611. What is the linear approximation for $f(x) = \cos x$ near $x = \frac{\pi}{3}$?

- (A) $\frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right)$ (B) $\frac{1}{2} - \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right)$ (C) $\frac{\sqrt{3}}{2} + \frac{1}{2}\left(x - \frac{\pi}{3}\right)$ (D) $\frac{\sqrt{3}}{2} - \frac{1}{2}\left(x - \frac{\pi}{3}\right)$ (E) $\frac{\sqrt{3}}{2} - \frac{1}{2}\left(x + \frac{\pi}{3}\right)$

211. [Calculator] When $x = 3$, the equation $2x^2 - y^3 = 10$ has the solution $y = 2$. When $x = 3.04$, $y =$

- (A) 1.6 (D) 2.14
 (B) 1.96 (E) 2.4
(C) 2.04

261. [Calculator] If $f(x) = 4^x - x^4$, the tangent to the curve is parallel to the secant through $(0,1)$ and $(4,0)$ for $x =$

- (A) 1.037 (D) **1.284 and 3.432**
 (B) 1.284 (E) 1.037 and 3.998
 (C) 3.998

281. [Calculator] To how many places is the symmetric difference accurate when it is used to approximate $f'(0)$ for $f(x) = 3^x$ and $h = .001$?

- (A) 1 (D) 4
 (B) 2 (E) **More than 4**
 (C) 3

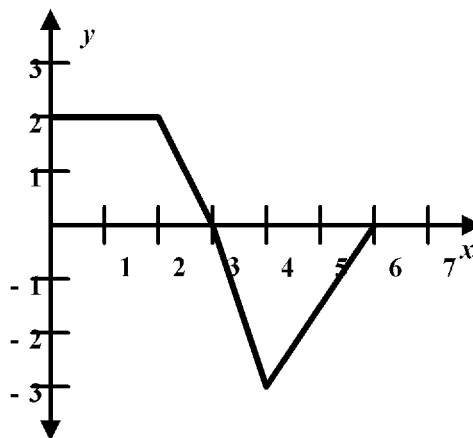
428. What is the best approximation for $\tan 61^\circ$?

- (A) 1.592 (D) 1.872
 (B) 1.662 (E) 2.012
(C) 1.802

433. What is the best approximation for $(9.99)^4$?

- (A) 9840 (D) 10040
 (B) 9920 (E) 10080
(C) 9960

588.



The graph of $f'(x)$ is shown above. If $f(1) = 4$, then the local linearization of f at $x = 1$ is $f(x) =$

- (A) $2x + 1$ (D) $2x + 4$
(B) $2x + 2$ (E) $2x - 1$
 (C) $2x + 3$

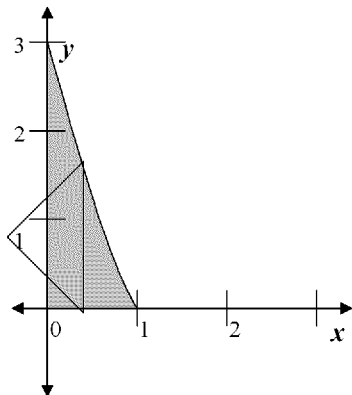
608. What is the linear approximation for $f(x) = xe^x$ near $x = 0$?

- (A) 1 (D) $-x$
 (B) -1 (E) Zero
(C) x

III. INTEGRALS
5. Part 2 Questions

A. Part 2 Questions
1. Part 2 Questions

1246.



The base of a solid S is the shaded region in the xy -plane enclosed by the x -axis, the y -axis, and the graph of $y = x^2 - 4x + 3$, as shown in the figure above. For each x , the cross section of S perpendicular to the x -axis at the point $(x, 0)$ is an equilateral triangle with one side lying in the xy -plane.

- (a) Find the area of the triangle as a function of x .
 (b) Find the volume of S .

$$(a) A = \frac{y^2 \sqrt{3}}{4} = \frac{\sqrt{3}(x^2 - 4x + 3)^2}{4}$$

$$(b) \frac{19\sqrt{3}}{30}$$

1249. Let R be the region enclosed by the graphs of $y = x^{\frac{2}{3}}$ and $y = \frac{1}{2}x$.

Set up and integrate an integral expression in terms of a single variable for the volume of the solid generated when region R is revolved about the x -axis.

$$\pi \int_0^8 x^{\frac{4}{3}} - \frac{x^2}{4} = \frac{256\pi}{21}$$

1257. John is playing a game in which he keeps choosing random numbers from zero to one until the sum is greater than 1. Construct an integral to represent the probability that John finishes the game after two repetitions.

$$\int_0^1 x^2 dx$$

1255. Let R be the region in the first quadrant enclosed by the hyperbola $y^2 - x^2 = 16$, the y -axis, and the line $y = 6$.

- (a) Find the volume of the solid generated by revolving R about the y -axis.
 (b) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved the line $y = -2$.

$$(a) \frac{56}{3}\pi$$

$$(b) V = 2\pi \int_4^6 (y+2)\sqrt{y^2-16} dy$$

1258. Let A be the region bounded on the top by $y = \sin x$, the bottom by $y = x^2$ and by the x -axis.

- a) Find the area of A .
 b) Find the volume of the solid generated when A is revolved around the x -axis.

a) .1357

b) 0.27936

1262. The rate at which a disease is spreading is proportional to the product of the proportions of the population that has the disease and the proportion of the population that does not have the disease. When $t = 0$, 1% of the population is infected. After two years, 10% of the population is infected.

- a) Construct a differential equation to model the spread of the disease.
 b) Solve the differential equation from part a).
 c) What proportion of the population is infected when the disease is spreading the fastest?

a) $P'(t) = kP(t)(1 - P(t))$

b) $P(t) = \frac{1 + \sqrt{1 - .0396e^{1.104t}}}{2}$

c) .5

1296. A rope is falling off a flat surface such that the acceleration of the rope is equal to gy/L , where L is the length of the rope, g is a constant, and y is the amount of rope that has already off the table.

- a) Verify that the equation $y = A e^{-kt} + B e^{kt}$ ($k^2 = g/L$) represents the amount of the rope y that is off the table at time t .
 b) Find A and B , given that the rope begins at rest and with $y(0) = y_0$.

b) $A = B = y_0/2$

1. Series of Constants

1. Convergence and Divergence

1128. Which of the following is true about the sequence $\left\{ \cos \frac{n\pi}{2} \right\}$?

- (A) It converges to 1
- (B) It is monotonic
- (C) It is unbounded
- (D) **It is bounded**
- (E) It converges to Zero

1147. For what values of n is the series $\sum_{n=1}^{\infty} (2n^2 - 7n)$ increasing?

- (A) $n > 0$
- (B) $n \leq 1$
- (C) $n \geq 1$
- (D) $n \leq 2$
- (E) $n \geq 2$

1643. To what limit does the sequence $S_n = \frac{5+2n}{5^n}$ converge to as n approaches ∞ ?

- (A) **0**
- (B) ∞
- (C) Sequence does not converge.
- (D) 5
- (E) 1

85. What is the radius of the series

$$\sum_{n=0}^{+\infty} 2^n x^n$$

- (A) 1
- (B) 2
- (C) $\frac{1}{2}$
- (D) 4
- (E) $\frac{1}{4}$

87. What is the radius of the series

$$\sum_{n=0}^{+\infty} (-1)^n \frac{x^{3n}}{(3n)!}$$

- (A) ∞
- (B) $\frac{1}{3}$
- (C) 3
- (D) $\frac{1}{9}$
- (E) 9

1063. Which of the following series is convergent?

- (A) $\sum_{k=1}^{\infty} k^{-4/3}$
- (B) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$
- (C) $\sum_{k=1}^{\infty} k^{-1}$
- (D) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{k}}$
- (E) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[2]{k^7}}$

1064. Which of the following infinite series is decreasing?

- (A) $\sum_{n=1}^{\infty} \left(\frac{2^n}{1+2^n} \right)$
- (B) $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n} \right)$
- (C) $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1} \right)$
- (D) $\sum_{n=1}^{\infty} \left(3 - \frac{1}{n} \right)$
- (E) $\sum_{n=1}^{\infty} \left(\frac{1}{n} \right)$

1066. Which of the following series is convergent?

- (A) $\sum_{n=1}^{\infty} \frac{\ln k}{9k}$
- (B) $\sum_{n=1}^{\infty} \frac{1}{k+9}$
- (C) $\sum_{n=1}^{\infty} \frac{1}{1+9k^2}$
- (D) $\sum_{n=1}^{\infty} \frac{k}{1+9k^2}$
- (E) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{k+9}}$

1118. Which of the following is true for a harmonic series?

- (A) **It diverges**
- (B) It converges to zero
- (C) It converges to a finite sum
- (D) It is a positive series
- (E) It is an alternating series